Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Sound and Vibration 270 (2004) 137–147
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Investigation of the anisotropy of hemi-dodecahedron noise source radiation

W. Weise*

Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany

Received 19 August 2002; accepted 7 January 2003

Abstract

If loudspeakers with the radiation characteristics of point sources located in a reflecting plane are needed, a possible option is to use polyhedra cut in half. In contrast to the octahedron, the dodecahedron can be cut for this purpose only in such a way that reflection at the plane does not complement it to again form a regular dodecahedron. But by modifying the symmetry, the isotropy of the radiated sound field is degraded. The investigations described relate to the anisotropy of the radiated intensity in the far field in comparison with the octahedron and the original regular dodecahedron, both by expansion in terms of spherical functions and by boundary element calculations.

1. Introduction

If sound sources radiating uniformly into all directions are needed for acoustic measurements, not only cubes but also octahedra and dodecahedra with one loudspeaker in each face are used. The greater the number of faces of which the polyhedron is composed, the higher the frequencies up to which isotropic radiation is achieved. However, when loudspeakers of a specified radius are used, only a less compact polyhedron design can be realized with polyhedra of higher order. In other words, the radius of the inscribed sphere of the polyhedron increases with increasing number of faces at the same radius of the inscribed circle of the faces. Nevertheless, the improvement in isotropy exceeds and the higher complexity of, for example, a dodecahedron is worth it.

*Current address: Fachgruppe Elektronische Materialeigenschaften, Fachbereich Materialwissenschaften, Technische Universität Darmstadt, Petersenstrasse 23, 64287 Darmstadt, Germany.
E-mail address: weise@e-mat.tu-darmstadt.de (W. Weise).

doi:10.1016/S0022-460X(03)00487-5

For certain applications, however, loudspeakers with the radiation characteristics of point sources located in a reflecting plane are needed. These are used for the qualification of hemi-anechoic rooms according to ISO 3745, Annex A in particular. Here a point source must be located directly on the floor of the room so that it radiates isotropically into the hemi-anechoic room above it. An elegant realization of this source consists in a tube which is embedded in the floor and operated as a horn. This presupposes, however, that the floor has a suitable opening. If this is not so, it suggests itself to use 'halved' polyhedra resting on the reflecting plane which can be imagined as being complemented mirror-symmetrically by reflection at the plane. Obviously, however, only the octahedron can be divided along a symmetry plane so that no faces are divided. If, on the contrary, a hemi-dodecahedron is used with one loudspeaker being arranged on top and five loudspeakers laterally, the lateral faces are only slightly cut but together with its mirror image it is not a regular dodecahedron: In contrast to the reflected image, the second ring of five loudspeakers is rotated in case of the exact dodecahedron. This break of symmetry due to reflection has an influence on the radiation characteristics. In the following, this will be investigated in comparison to the regular dodecahedron and the octahedron. The sound radiation of the regular bodies has also been dealt with in Ref. [1], and the expansion in terms of spherical functions used there will also be used in the following.

2. Theory

The farfield distribution of the pressure of a particular wave field radiated by a sound source—expanded in terms of spherical functions—is given by

$$p = \frac{\exp(\mathrm{i}kr)}{kr}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}(-\mathrm{i})^l q_{l,m}\mathrm{Y}_{l,m}(\vartheta,\varphi), \tag{1}$$

where φ, ϑ, r are spherical coordinates and k is the wave number, the time dependence $\exp(-\mathrm{i}\omega t)$ with the angular frequency ω having been omitted. Now the source is spherically idealized, with the velocity distribution $A(\vartheta,\varphi)$ on its surface. The expansion coefficients $q_{l,m}$ which determine the contribution of the individual spherical functions $\mathrm{Y}_{l,m}(\vartheta,\varphi)$ to the angular distribution of the sound pressure then are obtained as follows:

$$q_{l,m} = \frac{\rho_0 c}{\mathrm{h}_l^{(1)\prime}(kR)}a_{l,m}, \tag{2}$$

where ρ_0 and c are the density and the sound velocity in air, R is the radius of the loudspeaker sphere and $\mathrm{h}_l^{(1)\prime}(x)$ the derivative of the spherical Hankel function $\mathrm{h}_l^{(1)}(x)$ with respect to x. The $a_{l,m}$ are the expansion coefficients of the velocity distribution of the source:

$$a_{l,m} = \int_0^{2\pi}\int_0^{\pi} A(\theta,\phi)\mathrm{Y}^*_{l,m}(\theta,\phi)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{3}$$

the asterisk standing for the complex conjugate. The derivation of (2) can be found, for example, in Ref. [2, Chapter 11.3], and is also summarized in the appendix. The $q_{l,m}$ are also the expansion coefficients for the surface velocity A, weighted with $\rho_0 c/\mathrm{h}_l^{(1)\prime}(kR)$. The expression $(kR\,|\mathrm{h}_l^{(1)\prime}(kR)|)^{-1}$ represents the square root of the radiation efficiency. Its dependence on kR and l

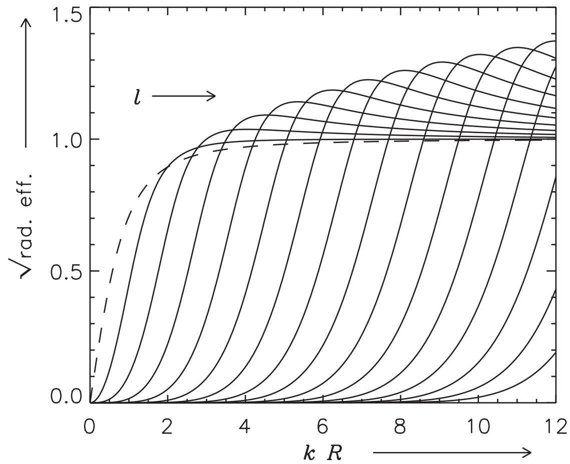

Fig. 1. Square root of the radiation efficiency of the spherical functions of index $l = 0\ldots15$, dashed: $l = 0$.

is shown in Fig. 1 for $l = 0, \ldots, 15$. It can be seen that a spherical function with index l will make a maximum contribution if $kR = \sqrt{l(l+1)}$ is approximately valid. For smaller kR its radiation efficiency for the function drops rapidly. This corresponds to the acoustic cancellation effect of bending waves on plates, which cannot radiate power if the bending wavelength is smaller than the wavelength of sound in the surrounding medium (see e.g. Ref. [3, Chapter 11]). Due to the finite size of the radiating sphere surface, the frequency-dependent onset of radiation in the case of a spherical function of a certain order is not, however, given by a step function as for infinite areas but is continuous. Conversely, the radiation efficiency at a frequency f of a spherical function with $l > 0$ is smaller than a specified value σ if the following statement is valid:

$$\frac{R}{[\sqrt{\sigma}\,(l+1)\cdot 1\cdot 3\cdot 5\ldots(2l-1)]^{1/(l+1)}} < \frac{c}{2\pi f}. \tag{4}$$

This can be shown by the asymptotic expansion of $\mathrm{h}_l^{(1)\prime}$. If l belongs to the spherical functions of lowest order involved in the loudspeaker arrangement besides that for $l = 0$, their radiation efficiency determines up to which frequency the radiation is isotropic. Therefore the design of a loudspeaker arrangement with isotropic radiation should aim to keep the radius as small as possible and to prevent spherical functions of low order from being involved in the expansion of the velocity distribution on its surface.

3. Point-source radiators in polyhedron faces

Let us consider polyhedra with point-source radiators in the centres of the faces. We will investigate the regular dodecahedron, the reflected hemi-dodecahedron described above with the reflecting plane parallel to one of the polyhedron faces, and the octahedron whose geometry is not changed through reflection.

First the ratio of the anisotropic share of the sound power in the total power will be investigated with respect to its frequency behaviour. It can be expressed by the coefficients $|q_{l,m}|^2$:

$$\frac{\sum_{l=1}^{\infty}\ \sum_{m=-l}^{l}|q_{l,m}|^2}{\sum_{l=0}^{\infty}\ \sum_{m=-l}^{l}|q_{l,m}|^2} = \frac{\overline{\langle p^2(t)\rangle - \langle p(t)\rangle^2}}{\overline{\langle p^2(t)\rangle}}. \tag{5}$$

Here $p(t)$ denotes the real sound pressure, $\langle\cdot\rangle$ means averaging over the spherical measuring surface which is situated in the far field, overbarred quantities have been averaged with respect to time. The quantity $\langle p^2(t)\rangle - \langle p(t)\rangle^2$ is the instantaneous value of the variance of the sound pressure on the measuring surface. For its determination, however, the sound pressures must be added in correct phase relation, which is of high metrological complexity. The quantity $\overline{\langle p^2(t)\rangle}$ is the average over the measuring surface of the mean-squared value of the pressure. In Fig. 2, the square root of Eq. (5)—a kind of standard deviation of the pressure—is plotted over $k\rho$ where ρ is the radius of the inscribed circle of the polyhedron faces. For the hemi-dodecahedron ρ is the distance between the lateral point-source radiators and the reflecting plane, as this limits the potential size of the loudspeakers. This assignment is based on the data for polyhedra with calotte tweeters for which the face dimensions must encompass the mounting ring usually surrounding the calottes.

Interestingly, Fig. 2 shows that as regards to isotropy of the radiation, the octahedron is similar to the hemi-dodecahedron, as for the former the symmetry is not broken and a more compact design can be chosen. The sphere radii of octahedron, hemi-dodecahedron, and dodecahedron, respectively, are given by: $R_{oct} = \sqrt{2}\rho$, $R_{h\text{-}dod} = 2\rho$, $R_{dod} = 1.618\rho$. The more compact design compensates for the composition of the octahedron which contains unisotropic spherical functions already of $l = 4$, but the hemi-dodecahedron only from $l = 5$ upwards. In the case of great tolerated anisotropy, the octahedron may even be better. The unisotropic spherical functions of lowest order needed for the regular dodecahedron are those with $l = 6$, which is the reason for its superior properties. In Table 1 the contribution $(\sum_m |q_{l,m}|^2/|q_{0,0}|^2)^{1/2}$ of the

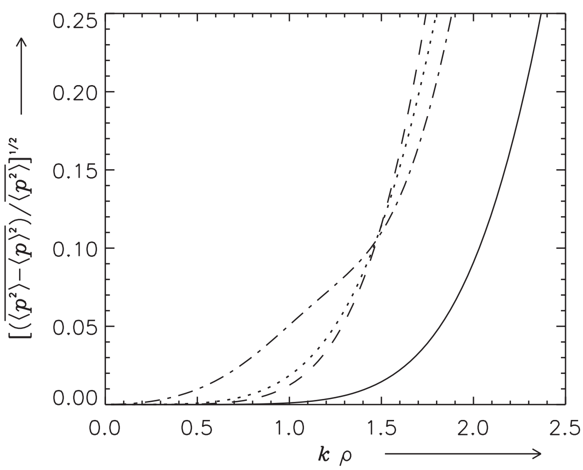

Fig. 2. Square root of the time-averaged variance of the sound pressure related to the averaged mean-squared value. Dodecahedron —, hemi-dodecahedron – – –, octahedron · · · ·, raised hemi-dodecahedron – · – · .

Table 1
Contribution of the groups of spherical functions to radiation for point-source radiators on polyhedron faces

l	Octahedron	Dodecahedron	Hemi-dodecahedron	Raised $0.05R$
0	1	1	1	1
2				0.08
4	1.53			0.13
5			1.11	1.01
6	2.27	2.39	1.59	1.60
7			1.40	1.58
8	0.88			0.24
9			1.32	1.07
10	2.98	1.66	0.96	0.67
11			0.08	0.60
12	2.08	2.93	2.65	2.70
13			1.29	1.49
14	2.50			0.65
15			1.36	0.87
16	3.49	2.58	1.93	1.76
17			1.94	1.85
18	1.99	3.19	1.99	2.03

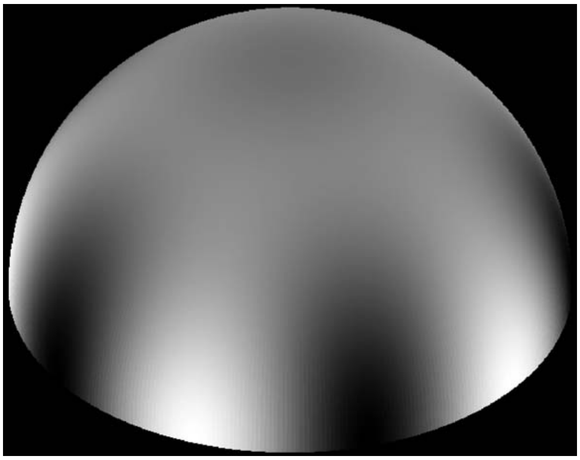

Fig. 3. Sound pressure level in the far field for the hemi-dodecahedron at $k\rho = 1.2$. Contrast range: 1.28 dB.

individual groups of spherical functions to the expansion of the surface velocity is represented for the three loudspeaker arrangements of point-source radiators up to $l = 18$. This quantity is independent of the alignment of the coordinate system on which only the assignment to the spherical functions of different m depends. If the reflecting plane is oriented along $\vartheta = \pi/2$, the spherical functions which are involved in the expansion of the hemi-dodecahedron must show a symmetry in the azimuthal direction which is divisible by 5, i.e., $m = 0, 5, 10, 15, \ldots$; another

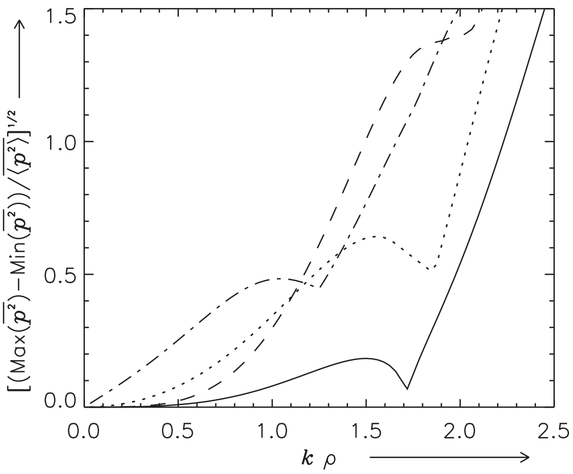

Fig. 4. Square root of the difference between maximum and minimum of the sound pressure square related to the averaged mean-squared value. Dodecahedron ——, hemi-dodecahedron – – –, octahedron · · · ·, raised hemi-dodecahedron – · – ·.

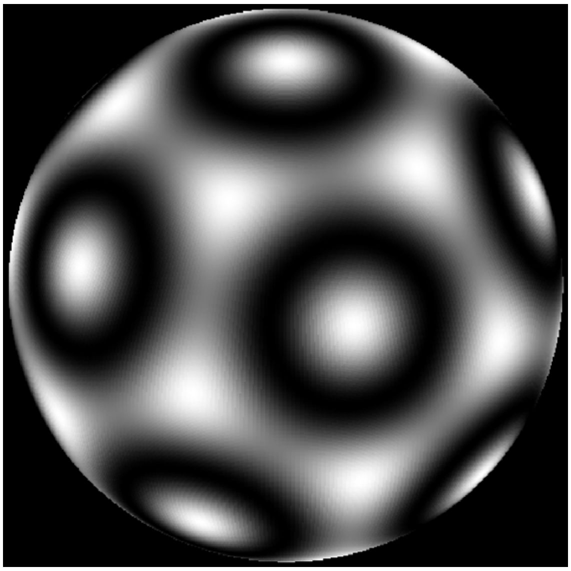

Fig. 5. Sound pressure level in the far field for the regular dodecahedron at $k\rho = 1.72$. Contrast range: 0.018 dB (local minimum).

necessary criterion is that $l + m$ must be even-numbered as exactly then the required mirror symmetry occurs.

The anisotropy of the hemi-dodecahedron is mainly relevant in areas close to the reflecting plane. This can be seen in Fig. 3 where the farfield sound levels are represented as grey-scale values. This appears to be plausible as the lateral point-source radiators and their mirror images are always very close to each other. The mathematical cause for this is that the lowest spherical functions contributing to the anisotropy, $Y_{5,\pm5}$, have a zero at $\vartheta = 0$ and their amount then increases monotonously up to $\vartheta = \pi/2$. Therefore the effects of an increase in the distance of the lateral point-source radiators from the reflecting plane have been investigated. For this investigation, all six radiators were kept on the surface of the inscribed sphere, i.e., only the centres of the lateral five loudspeakers were shifted. For small anisotropy this even results in degradation since now spherical functions from $l = 2$ are involved, as can be seen from Table 1, although a more compact design is possible for small shifts. The most compact design is achieved for a shift by $0.053R$ where the point-source radiators are centrally arranged between symmetry plane and upper edge of the side faces so that for the radius of the inscribed sphere $R = 1.828\rho$. For the distance $0.05R$ ($R = 1.828\rho$), the deviation from the isotropic sound pressure distribution is also shown in Fig. 2. In the domain of greater anisotropy, there is, however, an improvement over both the hemi-dodecahedron without additional distance and the octahedron.

Fig. 4 shows for the same loudspeaker types the square root of the difference between maximum and minimum of the square of the sound pressure related to the averaged mean-squared value. The depicted range of values spans about 3.5 dB of level difference. What is conspicuous is the dip of the anisotropy at certain frequencies. For the dodecahedron there is a dip almost to zero. The farfield sound level at the frequency in question is represented in Fig. 5 as a grey-scale image. Due to interference of the single radiators, an annular structure with only a small maximum amplitude is forming.

4. Boundary element simulations

Assuming cophasal motion of the loudspeaker membranes, to reach high isotropy, the use of loudspeakers with as large membranes as possible in general is more favourable than the use of

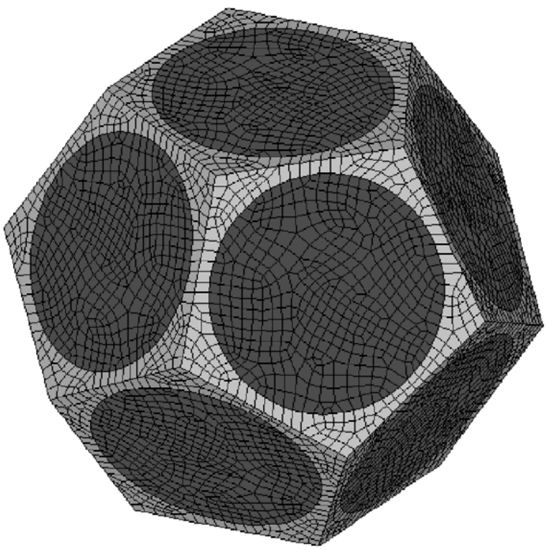

Fig. 6. Dodecahedron with BEM grid. The loudspeaker membranes are represented dark.

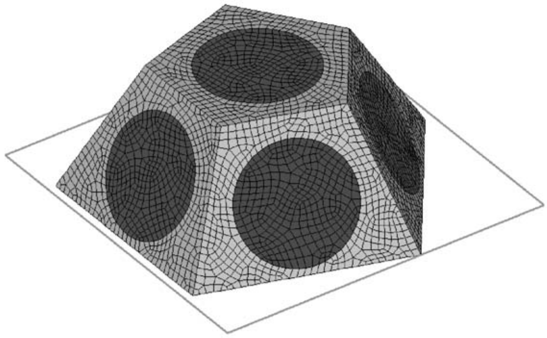

Fig. 7. Hemi-dodecahedron with BEM grid as well as symmetry plane.

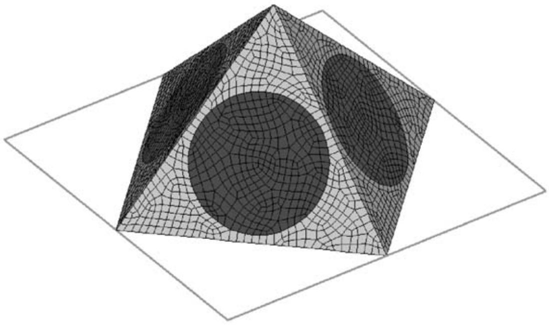

Fig. 8. Hemi-octahedron with BEM grid as well as symmetry plane.

small loudspeakers behaving more or less like point-source radiators. To investigate this in comparison to the above results for point-source radiators, realistic loudspeaker arrangements were simulated by the boundary element method (BEM). The geometry of the regular as well as the hemi-dodecahedron and of the octahedron can be seen in Figs. 6–8. For the purposes of the calculation, the radius of the loudspeaker membranes encompasses 90% of the radius ρ of the polyhedron faces. For the hemi-dodecahedron, as before, the centres of the loudspeakers are in the symmetry centre of the faces of the corresponding regular dodecahedron so that ρ is the distance between loudspeaker centre and symmetry plane. The hemi-dodecahedron whose lateral membrane centres have been moved away from the symmetry plane by $0.05R$ was also investigated. It can be seen in Fig. 9. The membranes which cover 90% of the maximum possible radius ρ cover a considerably greater portion of the faces than in Fig. 7. For the construction of a loudspeaker box it must, however, be taken into account that the interior space of the octahedron available for the individual loudspeaker is smaller, which may restrict the use of loudspeakers with high installation depth. The grid structures represented in the figures show the BEM grids used. As before, the pressure distributions in the far field were determined.

Fig. 10 shows for the three loudspeaker types the frequency dependence on $k\rho$ of the square root of Eq. (5), the ratio of the anisotropic share of the sound power in the total power. It is seen

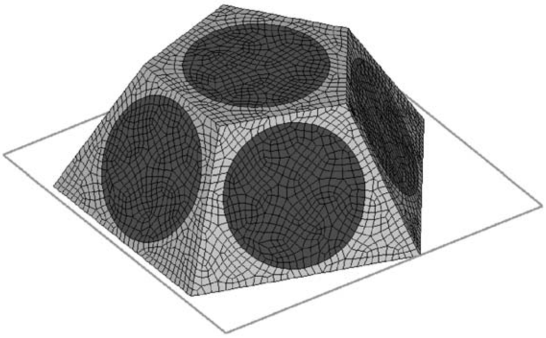

Fig. 9. Hemi-dodecahedron with shifted loudspeakers with BEM grid as well as symmetry plane.

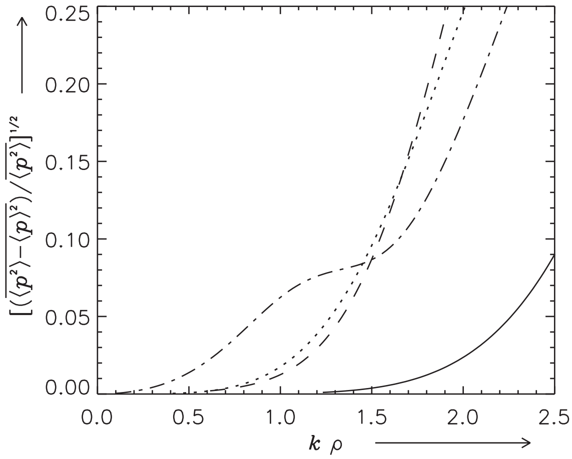

Fig. 10. Square root of the time-averaged variance of the sound pressure related to the averaged mean-squared value, BEM calculation. Dodecahedron —, hemi-dodecahedron – – –, octahedron · · · ·, raised hemi-dodecahedron – · – ·.

that compared with the previous results for point-source radiators, the regular dodecahedron has a much better isotropy. This can be explained by the fact that the membranes projected onto a spherical surface cover a great solid angle fraction. Octahedron and hemi-dodecahedron also show an improvement but this is only slight. For the hemi-dodecahedron with shifted membrane areas, this improvement is rather marked at higher frequencies but at lower frequencies the results for this source are even poorer than for the corresponding arrangement of point-source radiators.

Fig. 11 shows the curves of the maximum deviations for the four source types calculated as for Fig. 4. Dips of the isotropy obviously occur now at other frequencies. Here, too, it can be seen that hemi-dodecahedron and octahedron surpass each other in different frequency ranges as regards isotropic radiation. Unlike point-source radiators, the hemi-dodecahedron with shifted loudspeakers is even more favourable than the octahedron in the high frequency range. A clear difference results for all variants as compared to the regular dodecahedron.

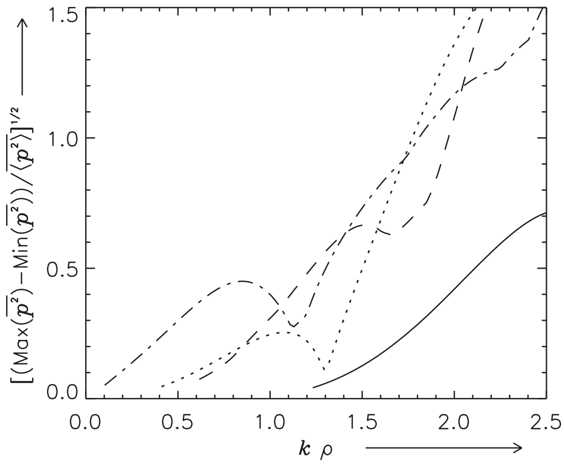

Fig. 11. Square root of the difference between maximum and minimum of the sound pressure square related to the averaged mean-squared value, BEM calculation. Dodecahedron —, hemi-dodecahedron – – –, octahedron · · · ·, raised hemi-dodecahedron – · – · .

5. Conclusions

If radiators derived from polyhedra are used as point sources in reflecting planes, the use of a hemi-dodecahedron of six loudspeakers cannot be generally recommended in the place of the less sophisticated hemi-octahedron with four loudspeakers. According to the frequency range, one or other can radiate more isotropically. The occurring maximum deviations between sound intensities in different directions depend strongly on the specific design. At certain frequencies, the maximum deviations of the sound intensity exhibit dips due to interference. For the hemi-dodecahedron, in the domain of higher anisotropy, as is typically tolerated for measuring loudspeakers in the field of sound emission, it proves to be suitable to displace the loudspeakers from their edge positions derived from the regular dodecahedron, as may be intuitively felt to be right.

Appendix

In spherical co-ordinates, the pressure field of an arbitrary outgoing wave is given by

$$p = \mathrm{i}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\mathrm{h}_l^{(1)}(kr)q_{l,m}\mathrm{Y}_{l,m}(\vartheta,\varphi). \tag{A.1}$$

For the radial component of the sound velocity v_r, the following relation is thus obtained:

$$v_r = \frac{-\mathrm{i}}{\omega\rho}\frac{\partial p}{\partial r} = \frac{1}{\rho c}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\mathrm{h}_l^{(1)\prime}(kr)q_{l,m}\mathrm{Y}_{l,m}(\vartheta,\varphi), \tag{A.2}$$

where $\omega/k = c$ was used. On the other hand, due to the completeness of the orthonormal system of the spherical functions, the radial sound velocity on the surface of the source is given by

$$v_r|_R =: A(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \int_0^{2\pi} \int_0^{\pi} A(\theta, \phi)\, \mathrm{Y}^*_{l,m}(\theta, \phi) \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\phi \, \mathrm{Y}_{l,m}(\vartheta, \varphi). \tag{A.3}$$

A comparison of Eqs. (A.2) and (A.3) yields Eq. (2). By asymptotic expansion of $\mathrm{h}_l^{(1)}(kr)$ in Eq. (A.1), Eq. (1) is obtained.

References

[1] V. Tarnow, Untersuchung der Schallabstrahlung von Lautsprecheranordnungen mit der Symmetrie regelmäßiger Körper, *Brüel & Kjær Technical Review* 4/74 (1974) 23–31.
[2] P.M. Morse, H. Feshbach, *Methods of Theoretical Physics, Part II*, McGraw-Hill, New York, 1953.
[3] L.L. Beranek, *Noise and Vibration Control*, McGraw-Hill, New York, 1971.